

Sect. 5.6

(19) Find $\cos(\frac{x}{2})$ if $\cos x = \frac{1}{4}$, $0 < x < \frac{\pi}{2}$

$$\cos(\frac{x}{2}) = \sqrt{\frac{1 + \cos x}{2}} \rightarrow \sqrt{\frac{1 + \frac{1}{4}}{2}} \rightarrow \sqrt{\frac{\frac{5}{4}}{2}}$$

$$= \sqrt{\frac{5}{8}} \rightarrow \frac{\sqrt{5}}{\sqrt{8}} \rightarrow \frac{\sqrt{40}}{8} \rightarrow \frac{2\sqrt{10}}{8}$$

$$= \frac{\sqrt{10}}{4}$$

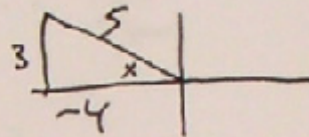
(20) $\sin(\frac{x}{2})$, given $\cos x = -\frac{5}{8}$, $\frac{\pi}{2} < x < \pi$

$$\sin(\frac{x}{2}) = \sqrt{\frac{1 - \cos x}{2}} \rightarrow \sqrt{\frac{1 - (-\frac{5}{8})}{2}} \rightarrow \sqrt{\frac{\frac{13}{8}}{2}}$$

$$= \sqrt{\frac{13}{16}} \rightarrow \frac{\sqrt{13}}{4}$$

(21) $\tan\left(\frac{x}{2}\right)$ given $\sin x = \frac{3}{5}$ $90^\circ < x < 180^\circ$

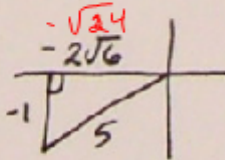
$$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1 + \cos x}$$



$$= \frac{\frac{3}{5}}{1 + \frac{-4}{5}} \rightarrow \frac{\frac{3}{5}}{\frac{1}{5}} \rightarrow \frac{3}{5} \cdot \frac{5}{1} = \boxed{3}$$

(22) $\cos\left(\frac{x}{2}\right)$ given $\sin x = -\frac{1}{5}$ $180^\circ < x < 270^\circ$

$$\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1 + \cos x}{2}}$$



$$= -\sqrt{\frac{1 - \frac{2\sqrt{6}}{5}}{2}} = -\sqrt{\frac{\frac{5 - 2\sqrt{6}}{5}}{2}} = -\frac{\sqrt{5 - 2\sqrt{6}}}{\sqrt{10}}$$

$$\boxed{\frac{-\sqrt{50 - 20\sqrt{6}}}{10}}$$

$$(33) \sin\left(\frac{x}{2}\right) = \sqrt{\frac{1 - \cos x}{2}}$$

$$\text{So } \sqrt{\frac{1 - \cos 40}{2}} = \sin\left(\frac{40}{2}\right) = \boxed{\sin 20^\circ}$$

$$(34) \cos\left(\frac{76}{2}\right) = \sqrt{\frac{1 + \cos 76}{2}} = \boxed{\cos 38^\circ}$$

$$(35) \tan\left(\frac{x}{2}\right) = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\tan\left(\frac{147}{2}\right) = \sqrt{\frac{1 - \cos 147}{1 + \cos 147}} = \boxed{\tan 73.5}$$

$$(51) \quad \sin^2 \frac{x}{2} = \frac{\tan x - \sin x}{2 \tan x}$$

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$$\left(\frac{1 - \cos x}{2} \right) =$$

↓

$$\frac{\frac{\sin x}{\cos x} - \frac{\sin x}{1} \cdot \frac{\cos x}{\cos x}}{2 \frac{\sin x}{\cos x}} \rightarrow$$

$$\frac{\frac{\sin x}{\cos x} - \frac{\sin x \cos x}{\cos x}}{2 \frac{\sin x}{\cos x}}$$

$$= \frac{\cancel{\sin x} (1 - \cos x)}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{2 \cancel{\sin x}}$$

$$= \left(\frac{1 - \cos x}{2} \right)$$

$$(53) \quad \frac{2}{1+\cos x} - \tan^2\left(\frac{x}{2}\right) = 1$$

$$\downarrow$$
$$\frac{2}{1+\cos x} - \frac{1-\cos x}{1+\cos x}$$

$$\frac{1+\cos x}{1+\cos x}$$

$$= 1$$

Ch 5 Test

- Solving Identities

- Finding exact value

e.g. Find $\cos(s+t)$ if $\cos(s) = \frac{3}{5}$ $\sin(t) = \frac{12}{13}$

Find $\cos 2\theta$ if $\sin \theta = \frac{3}{5}$

p. 231

#1-10, 13-16 (only \sin, \cos, \tan), 27, 29, 33, 34, 35, 37, 38, 49, 50,
52, 55, 57, 58

$$\frac{4}{\sqrt{6}-\sqrt{2}} \cdot \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}$$

$$\frac{4(\sqrt{6}+\sqrt{2})}{6-2} = \boxed{\sqrt{6}+\sqrt{2}}$$

$$\frac{-3+\sqrt{3}}{(3+\sqrt{3})} \cdot \frac{3-\sqrt{3}}{(3-\sqrt{3})}$$

conjugate

$$\frac{-9+6\sqrt{3}-3}{9-3} = \frac{-12+6\sqrt{3}}{6}$$

$$= \boxed{-2+\sqrt{3}}$$